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PHYSICS.—*The nitrogen thermometer scale from 300° to 630°, with a direct determination of the boiling point of sulfur.* ARTHUR L. DAY and ROBERT B. SOSMAN. Geophysical Laboratory, Carnegie Institution. To appear in full in the American Journal of Science.

1. PURPOSE AND PLAN OF THE INVESTIGATION

In our recent investigation of the fundamental high temperature scale with the gas thermometer[1] attention was chiefly directed to the temperature region above 1000°. The lower temperatures had been determined with considerable accuracy in an investigation undertaken in the Reichsanstalt[2] some years earlier (1900) and there was no reason to suspect in it any uncertainty of greater magnitude than the errors of observation determined at that time (2° to 3°). Our determinations (1911) of the temperatures below 1000° came out about 1° lower than those of the Reichsanstalt scales.

The situation from our viewpoint at the time of publication of our final results (1911) was accurately summed up in one of our closing paragraphs as follows:

[1] Preliminary publications:—Day and Clement: Physic. Rev., 24: 531, (Abstract) 1907. Day and Clement: Am. J. Sci. (4), 26: 405–463. 1908. Day and Sosman, Am. J. Sci. (4), 29: 93–161. 1910. R. B. Sosman: Am. J. Sci. (4), 30: 1–15. 1910. Final publication:—Day and Sosman: Carnegie Institution Pub. No. 157. 1911.

[2] Holborn and Day: Ann. Physik. (4), 2: 505–545. 1900. Am. J. Sci. (4), 10: 171-206. 1900.

The chief source of present uncertainty [in high temperature gas-thermometer measurements] is the temperature distribution over the surface of the bulb in an air bath. It would be possible to eliminate this error in the lower portion of the scale by substituting a liquid bath which could be stirred. In fact, this was done for temperatures below 500° in the earlier work of Holborn and Day, but has not, so far, been tried in the present investigation because of the relatively secondary importance of the lower temperatures to the ultimate purpose of the investigation (the study of silicates). For the higher temperatures, no sarisfactory liquid bath has been found.[3] (Publication of the Carnegie Institution of Washington No. 157, p. 115. 1911).

It was nevertheless thought wise to establish the identity and the magnitude of the error in these low temperature measurements which may have been due to this cause.

2. APPARATUS

No attempt will be made to describe in detail the gas thermometer and accessories used in this investigation. A full description of the apparatus, with illustrations, will be found in the publications to which reference has been made. No change whatever was made in the gas thermometer system for these measurements, except for the substitution, about the bulb, of a liquid bath, made up from potassium and sodium nitrates in eutectic proportions (55 per cent KNO_3, 45 per cent $NaNO_3$). This bath was introduced into the furnace bomb described in connection with the earlier apparatus in place of the platinum-wound furnace previously used. The insulating material was dry magnesia, as heretofore, and the bomb was water jacketed thruout to protect the manometer. The heating apparatus amounted, briefly, to one bath within another, that is, the tube containing the stirrer was continued across the bottom of the tank and upward about the bulb, which it fitted with but little clearance (10 mm.) in order to insure the very rapid circulation of a thin layer of liquid past the bulb, while the remainder of the bath remained at an approximately uniform temperature without. With this arrangement, no systematic temperature differences greater than the errors of observation of the thermoelement (0°.1) were observed.

[3] Professor Holborn has directed attention to the same limitations in our results at these temperatures (Ann. Physik., 35: 761–774. 1911).

3. METHOD OF PROCEDURE

In this apparatus temperatures were read simultaneously (1) upon the gas thermometer, (2) upon three thermoelements distributed at different points in the bath—one in a reëntrant tube extending to the center of the bulb, and one each at the top and bottom of its outside wall.

For the measurements at the benzophenone boiling-point, thermoelements of copper-constantan and of platinum-platinrhodium (Heraeus) were used, for the higher temperatures platinum-platinrhodium only. After a trial of the copper-constantan elements at the next higher temperature (zinc) evidence of permanent changes in their readings was obtained which was more than sufficient to offset their increased sensitiveness. They were accordingly abandoned at the higher temperatures.

Thru the courteous coöperation of the Bureau of Standards, a sensitive resistance thermometer in charge of Dr. Dickinson and Mr. Mueller of the Bureau was placed alongside the bulb during a part of the measurements, and later a similar instrument ingeniously constructed for the purpose by Dr. Dickinson was introduced into the reëntrant tube of the gas thermometer bulb itself. The resistance thermometer, which was easily sensitive to a few thousandths of a degree, revealed small temperature fluctuations (0°.05) in the rapidly circulating liquid outside the bulb but no systematic temperature differences. Within the reentrant tube the fluctuations were no longer felt.

With these precautions to guard against temperature differences about the bulb, temperatures were measured (1) at the boiling point of benzophenone, (2) at the melting point of zinc, (3) at the melting point of antimony.[4] The three thermoelements were then removed from the bath next day and placed in one or other of the following: in a vapor bath of boiling benzophenone, in an apparatus for determining the zinc melting point, or in a similar apparatus containing antimony; after which they were returned to the gas thermometer furnace for the verification of their read-

[4] The zinc and antimony were the same charges which were used in the previous investigation. The analyses may be found in Pub. No. 157, pp. 87 and 88; Am. J. Sci., **29**: 159.

ings. This series of operations constituted a set of observations as carried out in the tables which follow. Inasmuch as the gas thermometer was brought as close as practicable to the temperature of the points (benzophenone, zinc, etc.) selected as standards, the intermediaty rôle of the thermoelements was merely that of a transfer agent, in which rôle the individual properties of the thermoelements do not appear at all, provided the wires were originally homogeneous. The danger of contamination of the elements and consequent inhomogeneity is negligible at these temperatures. Even if such contamination had crept in it would have discovered itself in differences between the readings of the three elements with each change in the gradient, of which differences no trace was found.

By way of providing a strictly rigorous test of the accuracy of the transfer of temperature from gas thermometer bulb to the reference standards and its independence of the intermediary thermoelement, a special arrangement was devised in the case of zinc as follows: A steel bulb was made up with approximately the dimensions of the gas thermometer bulb and suspended in the same position in the nitrate bath. Enclosed in this bulb was the charge of zinc in its graphite crucible. In this crucible the thermoelement occupied the same position which it occupied in reference to the gas thermometer bulb, and all other conditions were, of course, identical. The zinc melting points were determined in this way, i.e., in a nitrate bath in which there were no measurable temperature differences in the region about the melting zinc and with the temperature gradient along the thermoelements identical with that surrounding the gas thermometer bulb itself.

4. BOILING POINT OF SULFUR

Finally an attempt was made to establish one temperature in this region from which the intermediary thermoelement should be completely eliminated. The gas thermometer bulb itself was immersed in the vapor of boiling sulfur. For this determination the nitrate bath was replaced by an appropriate sulfur boiling-point apparatus, all other conditions remaining the same. In building this apparatus, the experience of the Bureau of Stand-

ards was utilized for the most part. To this design certain modifications suggested by the unpublished work of Prof. G. A. Hulett of Princeton were added by way of rendering the determination as far as practicable independent of particular experimental conditions employed.

Heat was supplied electrically from a coil of high resistance wire about the sulfur tube, the coil ending about 2 cm. below the surface of the liquid sulfur (Bureau of Standards usage). An independent coil surrounded the vapor region, separated from it by an annular air space of about 1 cm. (Hulett). The bulb was surrounded first by a shield of sheet aluminum (Bureau of Standards) with holes near the top and bottom to permit the free circulation of the sulfur vapor, and a hole in the center of the bottom to permit the escape of liquid sulfur which chanced to condense on the shield. The shield afforded protection against any direct interchange of radiation with the furnace or with the boiling liquid, and its steep conical roof diverted the condensing liquid sulfur away from the bulb. Subsequently, with the purpose of varying these conditions, the aluminum shield was replaced by another of similar form but of glass (Hulett) and of somewhat smaller diameter. This was suspended from the conical aluminum roof of the first shield, which now overhung the side walls by several millimeters, with the effect that liquid sulfur condensing upon the cone could drip from the overhang instead of running down the side wall past the bulb. The radiation conditions were also radically altered by the substitution of glass for aluminum both around the bulb and below it.

Further variation was provided by changing the current in the two heating coils. Variations of some 40 per cent in the main coil about the boiling liquid were tried and the upper coil was varied from zero (Bureau of Standards usage) to nearly 40 per cent of the current in the main coil. Or, in other terms, the 1 cm. air jacket about the vapor was varied in temperature from the normal gradient (without heat in the upper coil) to a temperature equal to that of the sulfur vapor itself.

None of these changes produced any measurable change in the temperature of the sulfur vapor as recorded by the gas ther-

mometer, provided enough heat was supplied to fill the tube with vapor. During some of the measurements, the vapor escaped freely between the glass tube and aluminum cover and burned there.

TABLE I

GAS THERMOMETER MEASUREMENTS IN NITRATE BATH

DATE 1911	SERIAL NO.	p'	p	t	ϵ	ϵ AT FIXED POINT	TEMPERATURE OF FIXED POINT
Benzophenone, boiling point							
October 18	2	1023.36	1024.47	306.97	2379.8	2369.2	305.82
20	6	1020.60	1021.69	305.37	2364.7	2369.2	305.86
30	11	1053.07	1054.18	304.91	14944[1]	14996	305.82
November 1	14	1054.84	1055.97	305.91	15002[2]	15005	305.96
Mean							305.87
Zinc, melting point							
October 18	3	1216.27	1218.05	418.78	3429.4	3433.7	419.24
20	5	1215.85	1217.61	418.53	3427.1	3433.7	419.24[1]
November 4	16	1256.96	1258.76	419.32	3432.2	3433.7	419.48[2]
17	25	1256.54	1258.35	419.18	3432.9	3433.7	419.27
18	27	1255.85	1257.69	418.80	3429.0	3433.7	419.31
18	28	1254.96	1256.80	418.29	3423.8	3433.7	419.36
Mean							419.28
Antimony, melting point							
November 10	19	1627.10	1630.58	629.66	5525.8	5527.7	629.84
10	20	1627.09	1630.57	629.66	5525.2	5527.7	629.90
10	21	1626.96	1630.44	629.58	5524.2	5527.7	629.92
14	23	1625.08	1628.59	628.54	5515.5	5527.7	629.73
Mean							629.85

[1] Copper-constantan thermo-element.

[2] Wide temperature variation on manometer. This value is omitted from the mean.

5. EXPERIMENTAL RESULTS

In Table I, column 3, P' is the measured pressure of the gas in mm. of mercury at 0°; the application of the correction for the "unheated space" gives the pressure P of column 4; t (column 5) is the gas thermometer temperature in the nitrate bath, and e (column 6) the corresponding electromotive force of the thermo-element in microvolts; e (column 7) is the electromotive force of the same elements in benzophenone, zinc and antimony respectively, and the final column contains the corresponding gas thermometer temperatures.

Table II contains the measurements made in the sulfur vapor bath. p' and p represent the gas thermometer pressures corrected as before, t the resulting temperature, followed by the barometer reading reduced to sea level at lat. 45°, and the boiling temperature reduced to 760 mm. pressure.

TABLE II

DIRECT MEASUREMENT OF BOILING POINT OF SULFUR

DATE—1912	SERIAL NO.	p'	p	t	BAROMETER	t AT 760 MM.
January 31	49	1298.77	1301.07	443.21	746.3	444.45
	50	1298.83	1301.16	443.26	747.0	444.43
	51	1298.84	1301.17	443.27	747.0	444.44
February 1	52	1299.14	1301.44	443.42	748.4	444.47
	53	1299.14	1301.44	443.42	748.2	444.48
February 3	55	1299.70	1302.00	443.73	752.0	444.45
	56	1299.50	1301.81	443.62	751.4	444.40
	57	1299.55	1301.87	443.66	751.0	444.47
Mean						444.45

In Table III are brought together all the gas thermometer determinations of the boiling point of sulfur since 1890 with the necessary information for an intelligent comparison of the determinations. Column 5 contains the initial pressure of the gas used and column 6 the original value published by the author with a reference to the place of publication. Two of these determinations were subsequently corrected by the authors themselves. These corrections (with the references) are given in column 7.

TABLE III

GAS THERMOMETER DETERMINATIONS OF THE BOILING POINT OF SULFUR SINCE 1890

DATE	AUTHOR	THERMOMETER	GAS	PRESSURE	ORIGINAL FIGURE	CORRECTED FIGURE	THERMODYNAMIC SCALE	NOTES
				mm.				
1890	Callendar and Griffiths	Const. pr.	Air	760	444.53[1]		444.91	Indirect thru Pt. res.therm.[7]
1902	Chappuis and Harker	Const. vol.	N_2	530	445.2[2]	444.7[3]	444.80	Indirect thru Pt. res. therm.
1908	Eumorfopoulous.	Const. pr.	Air	760	443.58[4]	444.55[5]	444.93	Direct.
1911	Holborn and Henning	Const. vol.	H_2	623	444.51[6]			
			He	612				
			N_2	625	(444.39)		444.51	Indirect thru Pt. res.therm.
1912	Day and Sosman	Const. vol.	N_2	502	444.45		444.55	Direct.

[1] Phil. Trans., A 181: 119–157. 1891.

[2] Trav. Mem. Bur. Int., 12. 1902.

[3] Ibid., p. 90.

[4] Proc. Roy. Soc., A 81: 339–362. 1908.

[5] Ibid., A 83: 106–108. 1910.

[6] Ann. Physik, 35: 761–774. 1911. The value given for N_2 is calculated from the authors' table of experimental results. They give in their paper only the final mean value on the thermodynamic scale.

[7] This determination, often quoted as direct, is in reality indirect. In his first investigation (Phil. Trans. 1887) Callendar showed that his parabolic formula represented within 1° the variation of resistance with temperature as determined by the constant pressure air thermometer to 600°. In his later work (Phil. Trans. 1891) he showed by a comparison of two resistance thermometers with the air thermometer, using sulfur merely as a constant temperature bath, that his original value of $\delta = 1.57$ would still represent the results for these thermometers within the limits of error (about ± 0°.3). This value of δ was then used to calculate the sulfur boiling point determined with several platinum thermometers in the usual form of sulfur boiling tube.

Inasmuch as these various determinations were made under somewhat different gas conditions, the results are not directly comparable without reduction to some common unit. The fairest comparison is afforded by reduction to the thermodynamic scale.

6. SUMMARY

The new gas thermometer temperatures which this investigation has given us are brought together in Table IV, expressed (column 2) in terms of nitrogen expanding at constant volume from an initial pressure of 760 mm., and (column 3) in terms of the thermodynamic scale. These values replace the corresponding temperatures published in our papers to which reference has been made.

TABLE IV

SUMMARIZED TABLE

POINT	TEMPERATURE		NOTES
	Constant volume $p_o = 1$ at	Thermodynamic	
Benzophenone (Kahlbaum) boiling point at 760 mm	305.85	305.9	Transferred by platinrhodium and copper-constantan thermo-elements.
Cadmium, melting point	320.8	320.9	Interpolated.
Zinc, melting point	419.3	419.4	Transferred by thermo-elements.
Sulfur, boiling point at 760 mm	444.4	444.55	Direct.
Antimony (Kahlbaum), melting point	629.8	630.0	Transferred by thermo-elements.
Aluminum, melting point	658.5	658.7	Interpolated.

Finally, a comparative table (Table V) is added showing in terms of the same (thermodynamic) scale a comparison of our results with those obtained by Holborn and Henning in the latest work published from the Reichsanstalt.[5]

[5] Holborn and Henning. 1911. Loc. cit.

TABLE V

COMPARATIVE TABLE. THERMODYNAMIC SCALE

POINT	HOLBORN AND HENNING 1911	DAY AND SOSMAN 1912
Benzophenone	305.9	305.9
Cadmium	320.9	320.9
Zinc	419.4	419.4
Sulfur	444.5_3	444.5_3
	Holborn and Day	
Antimony	630.6	630.0

PHYSICS.—*Comparison of the resistance thermometer scale with the nitrogen scale from 300° to 630°.* H. C. DICKINSON and E. F. MUELLER, Bureau of Standards.

The temperature scale which at presents serves as the basis for the work of the Bureau of Standards in the interval 100° to 600°C. is the scale of the platinum resistance thermometer as defined by the work of Waidner and Burgess: *Platinum resistance thermometry at high temperatures.*[1] The resistance thermometer scale was established by assigning to the temperature of the sulfur boiling point the value 444°.70 at normal pressure, as the best value available at the time. On this resistance thermometer scale the following temperatures, among others, were found:

Boiling point of benzophenone	306°.02
Boiling point of sulfur (by definition)	444°.70
Freezing point of antimony	630°.71

A direct comparison of the temperatures cale as defined above, and the temperature scale of the nitrogen gas thermometer previously described by Day and Sosman, was suggested, and as such a comparison seemed very desirable we undertook the resistance thermometer measurements. Two thermometers were used, one "D" of the ordinary Callendar type which had been in use in the laboratory, and the other "R" constructed especially for this work. The latter was made with the lower part bent into the

[1] Bull. Bureau Standards, 6: 149–230. 1910. Reprint No. 124. 7: 3–11. 1910. Reprint No. 143.

form of a U so that the resistance coil could be placed in the reentrant tube of the gas thermometer, occupying the same position as had the thermocouple in the measurements of Day and Sosman. These two resistance thermometers represent the temperature scale already referred to. The thermometers were compared in the fused salt bath with the gas thermometer, which was operated by Messrs. Day and Sosman[2]. The points at which comparisons were made were approximately 306°, 445°, and 630°. After the observations in the salt bath the constants of the resistance thermometers were verified by recalibration. Two series were made with each thermometer. The results will be seen from the tables below:

TABLE I

Calibration of Thermometer D		*Calibration of Thermometer R*	
R_0	26.6733	R_0	4.8321
F.I.	10.3171 ± 0.0002	F.I.	1.8892 ± 0.0001
δ	$1.520 \pm 0.000_4$	δ	$1.503 \pm 0.000_3$

Eight determinations of each constant were made.

The figures indicate an accuracy of about 0°.01 in the determination of the fundamental interval and about 0°.03 in the determination of the boiling point of sulfur (440.70 in above) from which δ is computed, and show that either one of the thermometers adequately represents the resistance thermometer scale of Waidner and Burgess.

The results of the comparisons in the salt bath with the gas thermometer are shown in table II.

The figures in the second column are the serial numbers assigned to the gas thermometer observations. In the fourth column is given the temperature in the salt bath as indicated by the resistance thermometer. In the next three columns are the nitrogen thermometer data: p' is the original measured pressure in the nitrogen thermometer bulb in millimeters of mercury at 0°; the application of the correction for the "unheated space" gives the pressures (p) of column six; and in column seven is the temperature. From the differences in the eighth column and the values

[2] This Journal, 2: 171. 1912.

given on page 175, the following temperatures on the nitrogen scale of Day and Sosman are found:

B.P. of benzophenone ..... 305°.84
B.P. of sulfur ..... 444°.28
F.P. of antimony ..... 629°.75

TABLE II

DATE 1911	SERIAL NO.	RESISTANCE THERMOMETER USED	TEMPERATURE BY RESISTANCE THERMOMETER	NITROGEN THERMOMETER DATA			RESISTANCE MINUS GAS
				p'	p	t	
			Comparisons at 306°				
12-7	31	D	304.92	1052.58	1053.71	304.72	+0.20
	32	D	305.48	1053.60	1054.73	305.28	+0.20
12-21	41	R	306.72	1055.77	1056.92	306.52	+0.20
	42	R	306.91	1056.49	1057.64	306.93	−0.02*
							+0.18
			Comparisons at 445°				
12-7	33	D	446.55	1304.21	1306.22	446.06	+0.49
	34	D	446.54	1304.25	1306.26	446.09	0.45
12-9	36	D	446.38	1304.19	1306.22	446.07	0.31
	37	D	446.21	1303.92	1305.95	445.92	0.29
12-21	39	R	447.15	1305.25	1307.29	446.70	0.45
	40	R	447.15	1305.37	1307.41	446.77	0.38
12-26	46	R	443.30	1298.44	1300.47	442.88	0.42
	47	R	442.92	1297.44	1299.47	442.30	0.62*
							+0.42
			Comparisons at 630°				
12-26	44	R	631.20	1627.95	1631.46	630.21	+0.99
	45	R	630.96	1627.61	1631.12	630.02	0.94
							+0.96

* Rapidly changing temperature, not given full weight in mean.

For the sake of comparison, these temperatures, reduced to the thermodynamic scale, and those found by Messrs. Day and Sosman[1] by transfer with thermocouples, and also by direct measurement with the gas thermometer in sulfur, are shown below:

[1] This Journal, 2: 171. 1912.

The agreement of the transfer methods is complete. The difference of 0°.17 between the transfer and the direct determination at the boiling point of sulfur may be due to (1) difference in the sulfur baths used, (2) an unusual accumulation of accidental errors, (3) a systematic error affecting all transfer measurements.

TABLE III

POINT	TRANSFER BY RESISTANCE THERMOMETER	DAY AND SOSMAN	
		TRANSFER BY THERMOCOUPLES	DIRECT
B.P. of benzophenone	305.89	305.9	
B.P. of sulfur	444.38		444.55
F.P. of antimony	629.93	630.0	

1. The two sulfur baths were compared side by side by means of thermometer "R." The Day and Sosman sulfur bath was operated by Dr. Sosman. Exploration with the resistance thermometer showed no temperature differences as large as 0°.05 within the aluminium shield which had been used with the gas thermometer. The temperature in this shield was found to be 0°.03 to 0°.04 lower than in the Bureau of Standards gas-heated apparatus, a difference which is hardly significant.

2. The accidental errors of either the gas thermometer or the resistance thermometer are not as large as the difference found. The three values found by transfer with the resistance thermometer as well as the two values found by transfer with the thermocouples, give the same value of δ (1.484) for the resistance thermometer "R."

3. The above agreement, together with the fact that the value of δ computed from the value 444.55 for the sulfur boiling point is 1.494, may be taken as indicating a possible systematic error affecting all the transfer values. On account of the absence of a direct transfer with the thermocouples, however, this cannot be considered as proven. The indirect determination of the sulfur boiling point would appear to have less weight than the direct determination, since the sulfur bath used for the latter was found to define the same termperature as that obtained in the conventional form of sulfur boiling apparatus.

The boiling point of sulfur. The evidence based on all the data at present available is that the best value for the temperature of the boiling point of sulfur at normal pressure, is 444°.6 to the nearest 0°.1 on the thermodynamic scale.

PHYSICS.—*On the deduction of Wien's displacement law.* EDGAR BUCKINGHAM, Bureau of Standards.

The least satisfactory step in deductions of this law is usually the treatment of the change of wave-length of perfectly diffuse radiation upon reflexion from a moving surface. This step may be taken in the following manner:

Let a closed evacuated shell, with walls which are perfectly but somewhat irregularly reflecting, be filled with perfectly diffuse, approximately monochromatic radiation of wave-lengths between λ and $\lambda + d\lambda$. Let $R_\lambda \, d\lambda . ds . dw$ be the amount of radiant energy which passes in unit time from the negative to the positive side of a small plane surface element of area ds, inside the shell, in directions comprised within a cone of the infinitesimal solid angle dw described about the normal to ds. For diffuse radiation, the "radiant vector" R_λ is the same at all points and in all directions.

Let M be a small plane piece of the shell wall of area s, and let it have a normal velocity βC outward, C being the velocity of light and β an infinitesimal. The rest of the shell wall remains at rest. If $T = \lambda / C$ is the period of the waves at a point fixed in space, their period of arrival at a point fixed on M, from an angle of incidence φ, is $T' = T/(1 - \beta \cos \varphi)$ and the effect of arrival at this angle is to increase the period in the ratio

$$r_a = 1 + \beta \cos \varphi \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

terms of higher orders in β being negligible. Similarly, a disturbance starting from a point on M with a period T' and propagated at an angle of reflexion ψ, suffers a further increase of its period, measured at a point fixed in space, in the ratio

$$r_d = 1 + \beta \cos \psi \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Our problem is to find the effect on the original period T, of all the arrivals and departures at all possible angles φ and ψ

between 0 and $\pi/2$, within a long time t; and we shall evidently have to evaluate a product of the form

$$r_a r_d \cdot r'_a r'_d \cdot r''_a r''_d \text{ etc.}$$

But since multiplication is commutative, if we treat all the arrivals by themselves, then all the departures, and finally multiply the two results together, we shall arrive at the same value as if we considered the effects alternately, as they occur in fact.

We start from the consideration that in a sufficiently long time, every element of the energy within the shell must undergo reflexion from M at any particular angles (φ, ψ) just as often as every other element, so that the number of times n that any element is affected in a given way within a long time t is the quotient of the total amount affected in that way by the total amount present within the shell to be affected.

Taking the arrivals first, the whole energy which strikes M at angles between φ and $\varphi + d\varphi$, within t, is $t \cdot R_\lambda \, d\lambda \, s \cos\varphi \cdot 2\pi \sin\varphi d\varphi$, and the total energy present within the shell of volume v is $v \cdot 4\pi R_\lambda d\lambda / C$, so that we have

$$n = \frac{Cts}{2v} \cos\varphi \sin\varphi \, d\varphi \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

All these n arrivals together increase the period in the ratio

$$r_a^n = (1 + \beta \cos\varphi)^n = 1 + n\beta \cos\varphi \ldots\ldots\ldots\ldots (4)$$

Inserting the value of n from (3) and noting that $\beta Cts = \Delta v$ is the infinitesimal increase of volume which has occurred during t, we have

$$r_a^n = 1 + \frac{1}{2} \frac{\Delta v}{v} \cos^2\varphi \sin\varphi \, d\varphi \ldots\ldots\ldots\ldots\ldots (5)$$

The effect of arrivals during t at all possible angles is found by taking the product of all the factors of this form from $\varphi = 0$ to $\varphi = \pi/2$ or, neglecting higher powers of β,

$$1 + \frac{1}{2} \frac{\Delta v}{v} \int_0^{\frac{\pi}{2}} \cos^2\varphi \sin\varphi \, d\varphi = 1 + \frac{1}{6} \frac{\Delta v}{v}.$$

Similar reasoning on the effect of departures at the angle ψ leads to the same final expression, and we have for the combined effect of all the alternate arrivals and departures in all possible directions,

$$\frac{T+\Delta T}{T} = \left(1+\frac{1}{6}\frac{\Delta v}{v}\right)^2 = 1+\frac{1}{3}\frac{\Delta v}{v}$$

or, in terms of the wave-length,

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{3}\frac{\Delta v}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

This equation holds for any value of λ, hence the interval $d\lambda$ changes in the same ratio as λ itself. The result is valid for any element of the shell which is small enough to be treated as plane, for we have not assigned to M any special properties different from those of the rest of the shell wall. Equation (6) may therefore be integrated into the form.

$$\lambda = \text{const} \times v^{\frac{1}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

We have treated R_λ as remaining perfectly diffuse, but this is not exact because the motion of M disturbs the diffuseness. The resulting error, however, may easily be shown to involve only terms of lower orders of magnitude than those retained—which are sufficient— and equation (7) remains valid. There is, also, no objection to making the long time t infinite, if we still make βCts an infinitesimal of the first order by making $\beta s = \text{const} \times t^{-2}$.

Equation (7) gives the effect on wave-length caused by change of volume of diffuse monochromatic radiation enclosed within a non-absorbing envelope. It may be put into words as follows: if the shell retains its form during expansion or contraction, the wave lengths change in the same ratio as the linear dimensions of the shell, and the whole system of waves and shell remains geometrically similar to itself.

In the extended form of this paper, to appear in the Bulletin of the Bureau of Standards, more details are discussed and the remaining steps required to complete the deduction of the displacement law are given.

PHYSICS.—*A new precision colorimeter.* P. G. NUTTING, Bureau of Standards.

It is well known that any color may be analyzed and specified in either of two different ways: (1) in terms of three primary components, red, green and blue (trichromatic analysis) or (2) in terms of wave length of dominant hue or its complementary and per cent white (monochromatic analysis). In mathematical terms, the color point may be located by either trilinear or by polar coördinates. Of the three elements of color, hue, tone and luminosity, hue and tone are determined with a colorimeter, luminosity with a photometer or a photometric part of the colorimeter.

Colors to be analyzed consist of light either emitted from some source, transmitted through some selective screen or reflected from some mat or semi-mat surface. Any of these colors to be analyzed may be either spectral or purple according to whether or not its dominant hue lies in the visible spectrum. Any analyzing colorimeter must then be applicable to emitted, transmitted or reflected light of either spectral or purple dominant hue.

A trichromatic analyzer, the Ives[1] colorimeter, has been in successful use for several years. A monochromatic method of analysis was devised and used by Abney[2] in a laboratory investigation as early as 1890. The colorimeter here described is a monochromatic analyzer designed to be a practical working instrument of wide range, high precision and of the utmost simplicity.

The advantage of monochromatic analysis lies in the elimination of the arbitrary reference standards (red, green and blue), readings being given directly in wave length and per cent white. Both methods involve the definition and adoption of some standard of white. The trichromatic method may be used for either spectral or purple hues indifferently, the use of the monochromatic method involves an interchange of two sources or arms in passing from spectral to purple hues. Both methods give readings varying somewhat with the observer involving in some cases,

[1] F. E. Ives: J. Franklin Inst., 421–3. 1907.
[2] W. de W. Abney: Color Measurements and Mixtures, pp. 162–6. 1891.

correction to the absolute color scale based on the average properties of a number of normal eyes.

The new colorimeter is so arranged that light of a pure spectral hue may be mixed with white light to match the unknown or, in the case of purples, it is mixed with the unknown to match white. The match is made with a Lummer-Brodhum photometer cube.

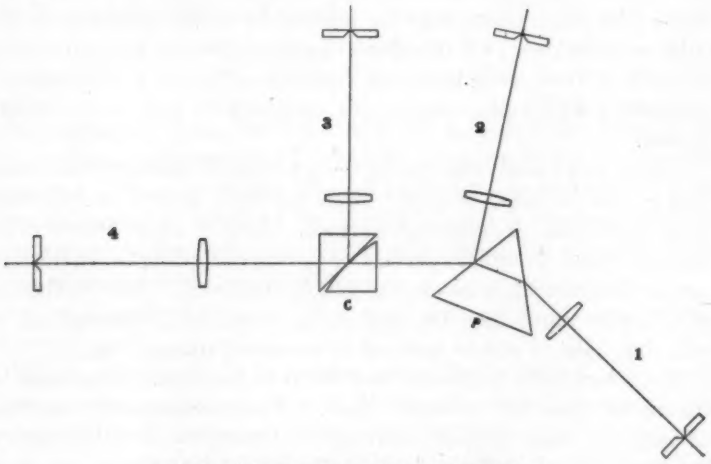


Fig. 1. Diagram of precision Colorimeter.

The figure is a diagram of the optical parts of the instrument. Collimator 1 is movable, all the remaining parts are fixed. Collimators 1 and 4 with prism *P* form an ordinary spectroscope with pin hole ocular. The white and unknown lights enter thru collimators 2 and 3, in direct or inverse order according as the match is to be made with the dominant hue or its complementary. In analyzing reflected light the necessary collimator is raised or rotated or even removed entirely if sunlight is used.

Wave lengths are varied simply by rotation of collimator 1. Intensities may be varied by (1) varying slit widths, (2) rotating sectors, or (3) by rotating one of a pair of nicols placed just inside each slit. The colorimeter now in use is provided with a bilateral slit on the first collimator while collimators 2 and 3 are provided with pairs of nicols.

After a match has been secured, the wave length of the dominant hue is either read from the position of collimator 1 or by throwing in a small hand spectroscope before the pin hole ocular of 4.

Intensities are determined by interposing between collimator 4 and photometer cube *C*, a white 180° (Whitman) rotating disk illuminated by a standard lamp. This gives the intensity in meter candles of each of the three component beams separately. Instead of this flicker photometer arrangement, a simple equality of brightness photometer may be used to determine the relative brightness of any two beams. To intercompare beams 1 and 2 for example, the top half of the objective of collimator 1 and the lower half of 2 are covered with a black card and then the width of slit 1 is varied until equality of brightness is secured. This slit width, compared with the original width used to secure a color match, gives the relative intensities of the two beams.

An experimental colorimeter of the type above described was assembled at the Bureau of Standards in the early Spring of 1911 and given a thoro test. Later special optical parts were ordered from Fuess and the instrument constructed in the Bureau shops. This new instrument has been in constant use at the Bureau since the first of January 1912 in routine tests and special research work. A patent dedicated to the public has been applied for.

Various problems arising in colorimetry are being investigated with the new instrument and will be reported upon in later papers. The sensibility of the instrument is, of course, that of the eye (chromatic and photometric) viewing objects directly. The precision attainable and the systematic errors to which the instrument and method are subject will be reported upon later.

GEOLOGY.—*Notes on the geology of the San Rafael Swell, Utah.* CHARLES T. LUPTON, Geological Survey. Communicated by Alfred H. Brooks.

The San Rafael Swell is an irregular elliptical dome, situated in east-central Utah between the Wasatch Plateau and Green River. It extends southwest from Price River almost to the Fremont or Dirty Devil River, a distance of about 80 miles.

G. K. Gilbert, in his monograph on "The Geology of the Henry Mountains, Utah," C. E. Dutton, in "The High Plateaus of Utah," and J. A. Taff, in Bulletin U. S. Geological Survey No. 285, refer briefly to this geologic feature. Other than the short statements in these reports the writer is unaware of any geological literature on this region.

During the field season of 1911, F. L. Hess spent two days on the east flank of the Swell on San Rafael River. References to that portion of the region are based on his observations. A party under the supervision of the writer, during the same season, mapped in detail the Upper Jurassic and Lower Cretaceous rocks along the west flank of the Swell in Castle Valley. In August, 1911, in company with W. R. Calvert, the north end of the region in the vicinity of Cedar Mountain or Red Plateau was visited. A little later accompanied by W. C. Mendenhall the writer entered the interior of the dome. At the close of the field season a reconnaissance trip was made with B. W. Clark to the north end of the Henry Mountains region, at which time the southern end of the San Rafael Swell was crossed. The observations made on these and other excursions form the basis of these notes.

The most prominent feature of the topography of this region is a series of buttes, mesas and "castles" which encircle an area, locally known as "Sinbad," which is 40 to 50 miles long and 10 to 20 miles in width. These fantastically eroded forms represent the outcrop of a gray massive cross-bedded Jurassic sandstone 800 feet thick. It is practicable to cross the Swell at only a few places on account of the almost impassable barrier formed by the sandstone rim. Nearly vertical scarps and canyon walls 300 to 500 feet in height are not unusual. The buttes and "castles," above referred to, are conspicuously shown on the San Rafael topographic sheet. Low "hogbacks," formed by resistant beds in the overlying strata, the tops of which produce dip slopes of varying extent depending on the inclination of the beds, encircle this belt of rugged topography. A view to the west from the interior of the Swell gives one the impression of looking up a very gently inclined varicolored stairway, the steps of which increase in height as the top, represented by the Wasatch Plateau, is

approached. Badland topography is common, especially near stream courses.

Structurally, the Swell is an elongated almost flat-topped dome extending northeast and southwest. The dip of the strata along the west flank is generally less than 10°, whereas that almong the east flank is as much as 70°. The strata in the interior of the dome are comparatively flat-lying, the principal line of flexure being near the east flank. Local minor domes were noted along the west side. Faults with displacements ranging up to more than 100 feet were observed in and near Cedar Mountain or Red Plateau at the north end of the region. Considerable faulting probably has occurred along the east flank.

Carboniferous (?), Triassic, Jurassic, and Cretaceous formations are well exposed. The lower part of the section described below was correlated with the section noted by Gilbert in the Henry Mountains regions, whereas the upper part is almost equivalent to the Book Cliffs section to the north. The lowest rocks exposed in this region are represented by a limestone probably of Carboniferous age noted by F. L. Hess near the San Rafael River on the east side of the Swell. Above the limestone there is a series of several hundred feet of sandstone and shale interbedded which probably are representatives of the Shinarump group (of Permian and Triassic age) and the Vermilion Cliff sandstone (of Triassic age). Unconformably (?) overlying these beds is a massive much cross-bedded gray sandstone 800 or more feet thick. This is the Gray Cliff sandstone (of Jurrassic age) of Gilbert's Henry Mountains section, which forms the striking topography referred to above and is, in all probability, the same as the White Cliff sandstone of the eastern Uinta and southern Utah sections of Powell. Conformably overlying this sandstone is a sequence of 1350 feet of reddish and gray sandstone, sandy shale, and thick beds of gypsum, which is approximately equivalent to the Flaming Gorge formation of Powell and corresponds closely with the Flaming Gorge formation as described by Gilbert in the Henry Mountains. Five hundred feet of conglomerate, sandstone and sandy shale of greenish drab color, overlying these strata correspond to the larger part of the Henrys Fork formation as identified by Gilbert in the

Henry Mountains, but probably should be classified with the Flaming Gorge formation of Powell. Sixty to one hundred feet of grayish sandstone (which, near Cedar Mountain, is replaced by conglomerate) and sandy shale with thin streaks of coal at the top and base, unconformably (?) overlie the conglomeratic strata just described and probably represent the Dakota sandstone. This sandstone corresponds to the uppermost part of Gilbert's Henrys Fork formation. The Mancos shale, about 4000 feet thick and consisting of three members, rests upon the Dakota(?) in apparent conformity. In places, however, a thin bed of conglomerate separates the Dakota(?) sandstone from the overlying shale. The Mancos outcrops in a monoclinal valley, the west part of which is known as Castle Valley. The lowest member consists of about 600 feet of bluish drab shale which is sandy in its lower and upper portions. Overlying this is a sandstone member approximately 500 feet thick containing coal beds near the top. This sandstone probably is equivalent to the Bluegate sandstone of Gilbert's Henry Mountains section and is represented at the north end of the Swell in the vicinity of Sunnyside Junction by a thin concretionary sandy formation. Overlying the sandstone is about 3000 feet of grayish drab shale which is sandy in its lower and upper parts. Conformably overlying the Mancos shale is the Mesaverde formation 1100 feet thick (in the vicinity of Emery) which caps the east scarp of the Wasatch Plateau. This formation consists mainly of sandstone with beds of sandy shale and coal intercalated.

Sills and dikes of basalt, which were noted at several localities near the south end of the Swell, extend as far north as Muddy or Curtis Creek.

ABSTRACTS

Authors of scientific papers are requested to see that abstracts, preferably prepared and signed by themselves, are forwarded promptly to the editors. Each of the scientific bureaus in Washington has a representative authorized to forward such material to this journal and abstracts of official publications should be transmitted through the representative of the bureau in which they originate. The abstracts should conform in length and general style to those appearing in this issue.

GEODESY.—*The effect of topography and isostatic compensation upon the effect of gravity.* JOHN F. HAYFORD and WILLIAM BOWIE. Special Publication No. 10, Coast and Geodetic Survey, pp. 130, with illustrations. 1912.

The introduction of isostasy in the determination of the figure and size of the earth from observed deflections of the vertical resulted in a marked increase in the accuracy of the values deduced. It was logical that isostasy should be considered in the reduction of gravity observations in order that a corresponding increase in accuracy of the shape of the earth might be obtained from these data.

A preliminary report on the reduction of 56 gravity stations in the United States, by the new method, was made to the International Geodetic Association at London and Cambridge in 1909 by Hayford. The present publication is a complete report on the reduction of 89 gravity stations in the United States.

By the new method a correction is applied at each station for the attraction of the topography of the whole earth and also a correction for the isostatic compensation of the topography in addition to the correction for the height of the station above sea-level. The compensation is assumed to be complete and uniformly distributed from the surface to a depth of 113.7 kilometers. This was the most probable limiting depth as determined by the first investigation of the figure of the earth and isostasy from measurements in the United States. The better value of 120 kilometers, obtained from the second investigation of the figure of the earth and isostasy, was not available at the time the tables were computed for the gravity reductions.

The methods used in applying the effect of the topography and its compensation are fully described and the necessary working tables are

given. In fact, all the data are given that are necessary for making reductions at any gravity station in the world.

The procedure at each of the gravity stations was to compute the theoretical gravity for the latitude of the station in question by the Helmert formula, correct this for the elevation of the station above sea level and then apply the correction for the topography of the world and its compensation. The resulting theoretical value is then compared with the actually observed value of gravity at the station. The difference between the computed and observed values is the anomaly which indicates a departure at that station from the assumed density of the earth's surface and of the computed excesses and defects of density in the crust to the depth of compensation.

A comparison is made of the new method anomalies in size and sign, with the anomalies given by the free air and Bouguer methods and the result is in practically every case in favor of the new method.

The 89 stations in the United States were arranged in groups with reference to their relation to topography. The groups are: 16 coast stations; 18 stations near the coast (within 325 kilometers); 27 continental stations not in mountainous regions; 16 stations in mountainous regions, below the general level; and 12 stations in mountainous regions, above the general level. The mean without regard to sign of the anomalies at 87 stations (two stations not considered) is 0.017 dyne. For the five groups mentioned above the corresponding means are 0.017, 0.020, 0.018, 0.012, and 0.014 dyne, of which no one is much above the general mean of all.

The means with regard to sign for the five groups are: − 0.004, + 0.002, + 0.002, − 0.002, and + 0.003 dyne. The general conclusion from the examination is that the anomalies by the new method show no relation to the topography either in sign or average magnitude. It is shown on the other hand that there are decided relations between the anomalies by the two older methods of reduction and topography.

It was found that there appears to be a relation between the surface geologic formation and the size and sign of the new method anomalies. The mean of the anomalies at seven stations on Pre-Cambrian formation is + 0.019 dyne, with regard to sign, and 0.026 dyne, without regard to sign. This indicates an excess of material in the crust in the vicinity of these stations. The means with and without regard to sign of the anomalies at 20 stations in the Cenozoic are − 0.011 and 0.021 dyne, respectively. This appears to indicate a deficiency in mass in the crust at these stations. It is shown that these anomalies are probably caused by

erroneous assumptions as to the density of the surface materials and also by a departure from a state of complete isostasy.

The mean without regard to sign of the new-method anomalies at the 89 stations in the United States is only 0.017 dyne. An anomaly of + 0.017 dyne would be produced by an excess of mass corresponding in amount to a stratum about 570 feet thick of density 2.67 (the mean surface density of the earth) with the station at the center of one surface of the disk and the disk of indefinite extent. An anomaly of − 0.017 dyne would be produced similarly by a deficiency of mass corresponding to a stratum about 570 feet thick. The gravity observations indicate, therefore, that the isostatic compensation is everywhere so nearly complete that the excesses and deficiencies of mass above the limiting depth of compensation correspond upon an average to a stratum only 570 feet thick. The average elevation of the surface of the ground in the United States is about 2500 feet, more than four times 570 feet.

The evidence furnished by the new method gravity anomalies in regard to the location and extent of the continuous areas of excess or deficiency of mass in the United States, that is, of under-compensation or of over-compensation, confirms and supplements that given by the observed deflections of the vertical previously considered and published by the Coast and Geodetic Survey in the second publication on the Figure of the Earth and Isostasy. W. B.

GEOLOGY.—*The Tertiary gravels of the Sierra Nevada of California.* WALDEMAR LINDGREN. Professional Paper U. S. Geological Survey No. 73. Pp. 226, with maps, sections, and illustrations. 1911.

This report attempts to trace the Tertiary history of the Sierra Nevada, by an examination of the Tertiary gravels and volcanic flows which cover a large part of their western slope. The gravels, as well known, have had, and still have, great economic importance, altho at present restrictions connected with the disposal of débris have put a stop to most of the hydraulic mining. The report is really a summary and includes observations made not only by the author, but by numerous other members of the Geological Survey.

In the early Tertiary age this region was a deeply eroded mountain range, this erosion having occupied the larger part of the Cretaceous period. Pauses in the erosion, when the topography had been reduced to gentle outlines, permitted deep rock decay and promoted the liberation of gold from its matrix. Renewed uplift quickened erosion and facilitated concentration. These conditions continued thruout Cre-

taceous and Tertiary time. Flucuations of the western shore line caused the streams at times to extend far into the areas now occupied by the Sacramento and San Joaquin valleys and at times to debauch upon flood planes high on the flanks of the range. Faulting movements with downthrow on the east side, probably beginning in Cretaceous time, had transformed an approximately symmetrical range into a monoclinal one with steep easterly slope. Gradually the mountains were worn down, the rivers flowed from low divides, meandering among longitudinal ridges, and the whole slope was covered by the dense vegetation of a damp semitropical climate.

Toward the end of Tertiary time rhyolite flows filled the valleys, covered the auriferous gravels, and new stream courses were outlined. Renewed disturbance began along the scarcely healed eastern breaks, resulting in a westward tilting of the main blocks, combined with normal faulting and subsidence of the blocks in western Nevada. In consequence of this disturbance the monoclinal nature of the range became strongly emphasized, and the streams immediately began to cut their beds deeper. At the end of the Tertiary, eruptions of andesites and their tuffs and breccias began in enormous volume and filled the Tertiary valleys to the rims. During the subsequent canyon-cutting epoch which belongs in part at least to the Quaternary period, the erosion has laid bare the old gravels and in most cases completely intersected the old valleys.

The first part of the book outlines the history of the valley border and describes the sedimentary and volcanic formations exposed at the western base of the range. The Tertiary drainage system and the prevolcanic surface is then described. The courses of the main rivers of Tertiary age draining the northern part of the range are traced in detail. Toward the southern part of the area the present drainage corresponds fairly well with that of the Tertiary period, the principal difference appearing to be that many of the Tertiary rivers flowed parallel with the range in longitudinal valleys which have been in most cases squarely intersected by the present Quaternary rivers. Feather River, one of the greatest of the present water courses, had no adequate representative in Tertiary time, but that part of the slope now belonging to it drained to the north.

Evidence is adduced by numerous profiles and sections of the character of the Tertiary topography, and of the present grade of the Tertiary stream channels. These profiles, by the infallible relation between extremely steep transverse grades and extremely flat longitudinal grades,

proves that a tilting uplift has taken place whereby the grades of the rivers flowing west have been considerably increased. The increase is so uniform as to show that the block tilted without deformation. The tilting of a marginal continental block 80 miles wide and 300 miles long, with inconspicuous deformation, must have required forces acting thru depths of miles.

In chapter 3 the fossils of the auriferous gravels are described, principally by Prof. F. H. Knowlton. Chapter 4 deals with the gold in the Tertiary gravels, its distribution, size, and value, as well as with the minerals accompanying the gold in the gravel. A brief account is given of methods of mining and of legislation concerning mining débris. An appendix gives a summary of the latest developments in the gold-dredging industry at the foot of the Sierra Nevada. W. L.

GEOLOGY.—*Alunite, a newly discovered deposit near Marysvale, Utah.* B. S. BUTLER and H. S. GALE. Bulletin U. S. Geological Survey No. 511. Pp. 64, pls. 3. 1912.

The alunite deposit described in this bulletin is located about 7 miles southwest of Marysvale, Piute County, Utah, on a spur of the Tushar range.

The deposit occurs filling a fissure vein cutting volcanic rocks. It outcrops in places for a distance of 3500 feet and has a maximum measured thickness of about 20 feet. Much of the vein material is nearly pure alunite, while in other parts it contains some quartz and kaolin.

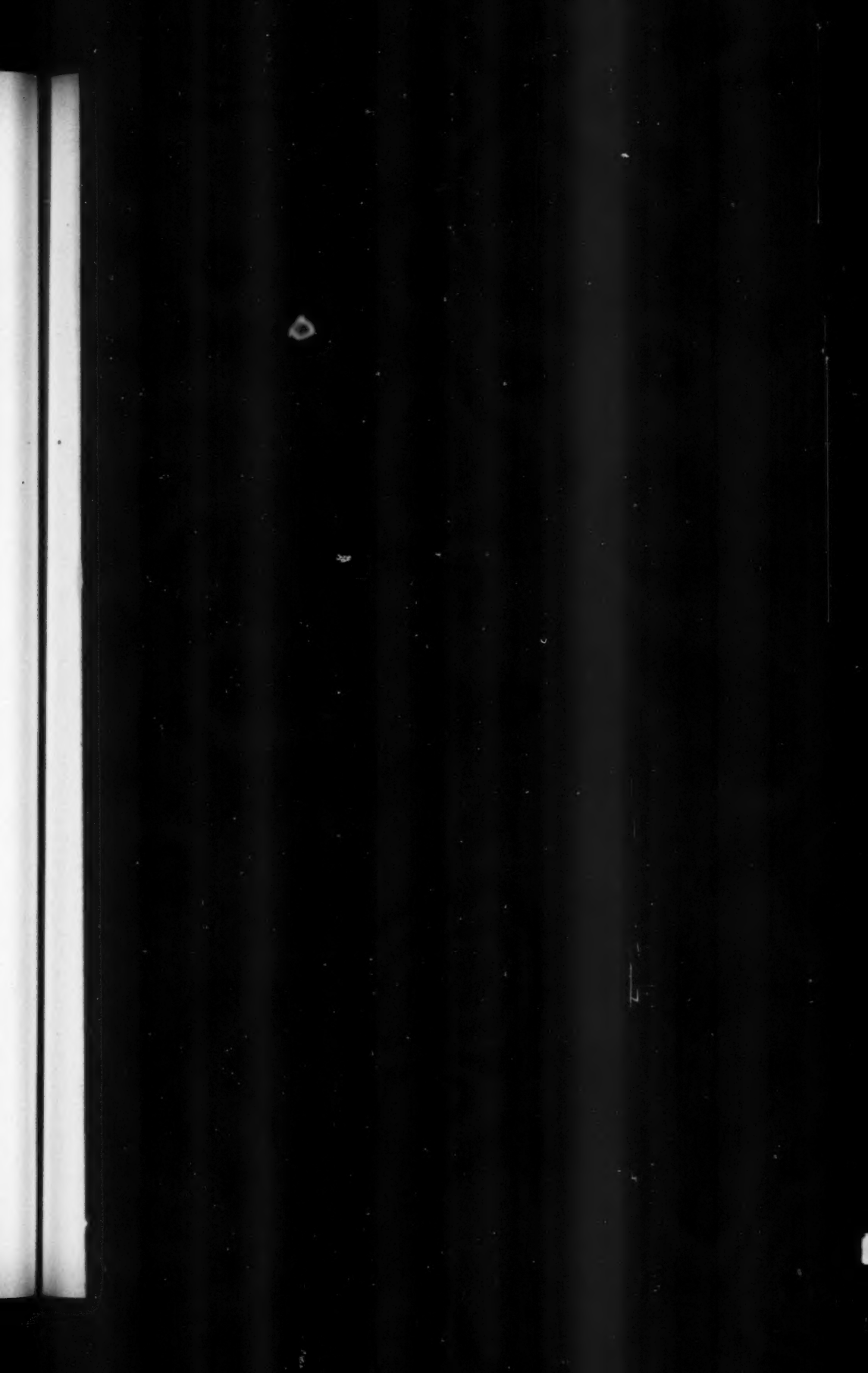
The bulletin also contains a brief description of other known occurrences of alunite both domestic and foreign and a review of commercial processes of treatment of this mineral. B. S. B.

PHYTOPATHOLOGY.—*The history and cause of the coconut bud-rot.* JOHN R. JOHNSTON, Bureau of Plant Industry. Bulletin 228. February 5, 1912.

A disease of coconuts called the bud-rot has long been known in Cuba, Jamaica, British Honduras, Trinidad, and British Guiana; it probably occurs in the Philippines, Ceylon, British India, German East Africa, and Portuguese East Africa The symptoms of this disease are the yellowing and falling of the leaves and the dropping of immature nuts. Eventually the middle folded leaves shrivel and bend over and the entire heart of the crown is involved in a vile-smelling soft rot. The tree may be killed within a few months after infection, and entire groves may be destroyed within two or three years. The cause of the disease in eastern

Cuba is an organism practically identical with *Bacillus coli* (Escherich) Migula. Inoculations into coconut seedlings with *B. coli* of animal origin gave infection similar to inoculations with the coconut organism. It is believed that birds and insects are carriers of this disease; stomatal infections are common on the young tissues. The study of the bud-rot disease of the coconut palm suggests that diseases of several other palms may be due to the same cause. No coconut district is secure from danger of infection of the bud-rot disease. The control of the disease will probably depend upon better sanitation and improved methods of cultivation. The cutting out and burning of all diseased tree tops and the removal of fallen leaves and nuts are especially necessary. It is believed that the bud-rot disease of coconuts does not occur in the island of Porto Rico. K. F. KELLERMAN.

PROGRAMS AND ANNOUNCEMENTS

PHILOSOPHICAL SOCIETY OF WASHINGTON

Saturday evening, April 6, 1912; Cosmos Club. MR. A. F. ZAHM: *Evolution of the Aeroplane; its Fundamental Features of English origin.*